A050231 Want: # of n-strings (trinary) with 3 or more As in a row & just one L.

~~4 → 0 → 1 → 0~~

$3^{5} = 243$
$\times 3$
729

~~Strategy~~
- enumerate all n-strings.
- count ones that contain ≥3 A's in a row.
- for each of those, count # of non-A spots.
- tally & analyze.

000

$n=3$ 0_1

$n=4$ $0_1, 1_4$

$n=5$ $0_1, 1_8, 2_{12}$

$n=6$ $0_1, 1_{12}, 2_{36}, \cancel{4}3_{32}$

$n=7$ $0_1, 1_{14}, 2_{72}, 3_{128}, 4_{80}$

$n=8$ ~~$0_1, 2_{108}, 5$~~
$0_1, 1_{16}, 2_{108}, 3_{320}, 4_{400}, 5_{192}$

$n=9$ $0_1, 1_{18}, 2_{144}, 3_{592}, 4_{1200}, 5_{1152}, 6_{448}$

$n=10$ $0_1, 1_{20}, 2_{180}, 3_{928}, 4_{2640}, 5_{4032}, 6_{3136}, 7_{1024}$

$n=11$ $0_1, 1_{22}, 2_{220}, 3_{1312}, 4_{4800}, 5_{10272}, 6_{12544}, 7_{8192}, 8_{2304}$

A_k = # ~~quads open~~ open spots count.

$n \to (n-3)_{\cancel{n-4}\; n-2},\ (n-4)_{(n-3)^2},\ (n-5)_{\frac{(n-4)^2}{(n-3)/2}},$

n=3 $\quad 0_1$

n=4 $\quad 0_1,\ 1_2$

n=5 $\quad 0_1,\ 1_4,\ 2_3$

$\cancel{0_1, 1_4, 2_3}$

n=6 $\quad 0_1,\ 1_6,\ 2_9,\ 3_4$

$q \longrightarrow (q-1)^2 \longrightarrow \dfrac{(q-2)^2 \cdot (q-1) \cdot \frac{1}{2}}{\cancel{(q-1)}}$

n=7 $\quad 0_1,\ 1_7,\ 2_{18},\ 3_{16},\ 4_5$

n=8 $\quad 0_1,\ 1_8,\ 2_{27},\ 3_{40},\ 4_{25},\ 5_6$

65 → 75

140 → 126

226 → 196

n=9 $\quad 0_1,\ 1_9,\ 2_{36},\ 3_{74},\ 4_{\boxed{75}},\ 5_{36},\ 6_7$

n=10 $\quad 0_1,\ 1_{10},\ 2_{45},\ 3_{116},\ 4_{165},\ 5_{\boxed{126}},\ 6_{49},\ 7_8$

Do it recursively!

n=11 $\quad 0_1,\ 1_{11},\ 2_{55},\ 3_{164},\ 4_{300},\ 5_{321},\ 6_{\boxed{196}},\ 7_{64},\ 8_9$

n=12 $\quad 0_1,\ 1_{11},\ 2_{66},\ 3_{220},\ 4_{480},\ 5_{666},\ 6_{567},\ 7_{\boxed{288}},\ 8_{81},\ 9_{10}$

n=13 $\quad 0_1,\ 1_{13},\ 2_{78},\ 3_{286},\ 4_{710},\ 5_{1197},\ 6_{1323},\ 7_{932},\ 8_{405},\ 9_{100},\ 10_{11}$

n=14 $\quad 0_1,\ 1_{14},\ 2_{91},\ 3_{364},\ 4_{1000},\ 5_{1952},\ 6_{2646},\ 7_{2416},\ 8_{1449},\ 9_{550},\ 10_{121},\ 11_{12}$

g = guys with one L not ending in A.

1

O $\xrightarrow{\text{"O"}}$ e

A $\xrightarrow{\text{"L"}}$ a - c

L $\xrightarrow{\text{"A"}}$ 0

$1 \to 3$

total - guys with one L.

f = guys with one L ending in ~~A~~A

$\xrightarrow{\text{"O"}}$ 0

$\xrightarrow{\text{"L"}}$ 0

$\xrightarrow{\text{"A"}}$ -f + g

n=3 e = 1

n=4 e = 3

$\boxed{\text{trace back e \& f \& g to find out where it's all going wrong}}$

↳ 200 instead of 199

e = guys with no L ending in AA

$\xrightarrow{\text{"O"}}$ 0

$\xrightarrow{\text{"L"}}$ 0

$\xrightarrow{\text{"A"}}$ F

OO
A O
L O
O L
A L
O A
A A
L A

[19, 2, 7, 5]

✓ ✓ ✓
3, 8, 19, ... 44 ?

AA OL's A

a b c d
[8, 1, 4, 2] $\xrightarrow{\text{"O"}}$ [a, 0, c, 0]

$\xrightarrow{\text{"L"}}$ [c, 0, 0, 0]

$\xrightarrow{\text{"A"}}$ [a - b, d, c-b, a - (b + d)]

e = # guys with one L ending in A.

8 + 4 + 7 19 +

OOO	OOL	OOA
AOO	AOL	AOA
LOO	OAL	LOA
OLO	AAL	OLA
ALO		ALA
OAO		OAA
AAO		LAA
LAO		

[43, 5, 13, 12]

7 + 0 +

guys with one L

(a - c)

[3, 0, 2, 1, 0]

O
A
L

c - b

e = guys with one L who end in A.

a - c

guys that have an L AND end in 2 A's.

LAA

$\xrightarrow{\text{"O"}}$ e

L

O

$\xrightarrow{\text{"L"}}$ e

guys with one L who ended in A.

$\xrightarrow{\text{"A"}}$ -e +

A/L/O

Suppose L : total (n-1) new such strings
prizers = # of guys in last one with 0 L's.

F(n) = # of n-length prize strings

$$F(n) = \overset{O}{F(n-1)} + \overset{A}{\underset{\text{\% not ending in "AA"}}{F(n-1)\cdot p}} + \overset{L}{\underset{\text{\% with no "L"s}}{F(n-1)\cdot b}}$$

O
A
L

$F(0) = 3$

$$F(1) = \overset{O}{3} + \overset{L}{2} + \overset{A}{3}$$

O	OO	OL	OA
A	AO	AL	AA
L	LO		LA

[total, ending in AA, zero L's, ending in A]

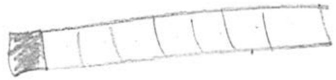
$$[3, 0, 2, 1] \xrightarrow{\text{"O"}} [+3, +0, +2, +0]$$

$$\xrightarrow{\text{"L"}} [+2, +0, +0, +0]$$

$$\xrightarrow{\text{"A"}} [+3, +1, +2, +2]$$

k blocks

$k=1 \to \binom{n-1}{1}$



$n=11$

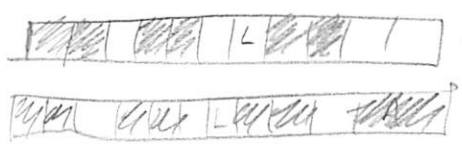
$\binom{n-2k+1}{k}$

$n-k-(k-1)$

$(n-2k)$

$n-2k+1$

$\times\, 2^{\#\text{ of tilable spots}}$



} → must be counted differently

check for $n=4$

1

4

7

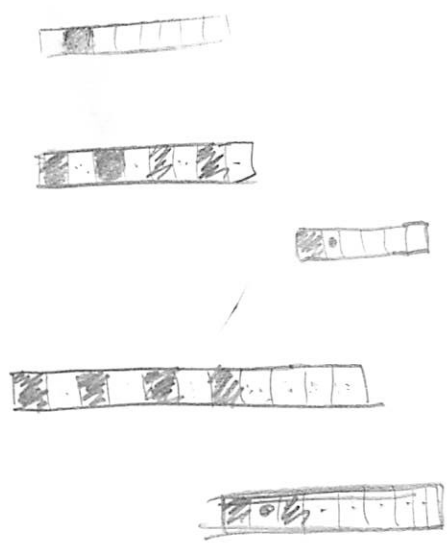
18

six possibilities:

- no A's whatsoever, no L's $\longrightarrow 1$
- no A's, one L $\longrightarrow n$
- single A's, no L's $\longrightarrow \displaystyle\sum_{k=1}^{\lceil n/2\rceil}\binom{n-k+1}{k}$
- single A's, one L $\longrightarrow \displaystyle\sum_{k=1}^{\lceil n/2\rceil}\binom{n-k+1}{k}(n-k)$
- double A's, no L's $\longrightarrow \displaystyle\sum_{k=1}^{\lceil n/3\rceil}\binom{n-2k+1}{k}$
- double A's, one L $\longrightarrow \displaystyle\sum_{k=1}^{\lceil n/3\rceil}\binom{n-2k+1}{k}(n-2k)$

k blocks

$k=1$ $\quad \binom{n}{1}=n.$



$n=11$

$\binom{n-k+1}{k}$

the double As can include other single A's!

$n-k+1$

$3^4 = 81$

-2 (three $\overset{A}{\text{in}}$ a row) $\boxed{\cdot 3}$ $- 1$ $\quad 81 - 13 = 68.$

$-\binom{4}{2} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot \cancel{2}} = 6$ (2 Ls) $\cdot \boxed{2^2}$

$-\binom{4}{3} = \frac{4 \cdot 3 \cdot 2 \cdot}{3 \cdot 2 \cdot} = 4$ (3 Ls) $\cdot \boxed{2}$

$-\binom{4}{4} = 1$ (4 Ls)

Consider.. combos of permus around each of these.

$81 - \overset{5}{\cancel{6}} - 24 - 8 - 1 \Rightarrow 42 = \boxed{43}$

$\boxed{\text{intersections}}$

Ls overlaps.

AAA$\overset{O}{\underset{A}{L}}$
→ double count
$\overset{O}{\underset{A}{L}}$AAA

$2 \cdot 2 = 4$

AAAA = 1

A A A _ _

AAA$\overset{2}{\uparrow}$O$\overset{3}{\uparrow}$O
OA
OL
LO
LA
$\boxed{LL}$

$n = 7$

$k = 5$

$n - k + 1$

_ _ _ _ _ _ _

17 3 or more As in a row

3 places $\cdot$ 2^2 others $= 12$

2 places $\cdot$ 2 others $= 4$

1 place = 1

k As n spots

$\boxed{3^{30}}$

Add back in all the n a's in L's.

O O O O
L A A A L L L
A A A A

k

the ends of n−k+1−2

− 3 A's in a row $= 2 \cdot 2 \cdot 3^{26} + 26 \cdot 2^2 \cdot 3^{25}$

− 4 As in a row $= 2 \cdot 2^1 \cdot 3^{25} + 25 \cdot 2^2 \cdot 3^{24}$

⋮

− 28 As in a row $= 2 \cdot 2^1 \cdot 3^1 + 1 \cdot 2^2$

− 29 As in a row $= 2 \cdot 2^1$

− 30 As in a row $= 1$

$\frac{2}{\cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot \cancel{1}}$

≥3 As in a row.

$81 - \binom{4}{2} \cdot 2^2 - \binom{4}{3} 2^{-1}$ ≥2 Ls.

$24 - 8 - 1$

A050231=?

$48 - 3 \boxed{-2}$

figure out some way to get 9 for a specific # of As. then sum from k=3 to n.

STEP 1

− 2 Ls $= \binom{30}{2} \cdot 2^{28}$

− 3 Ls $= \binom{30}{3} \cdot 2^{27}$

⋮

− 30 Ls $= \binom{30}{30} \cdot 2^0$

STEP 2

3 or more As.

All else are Os.

Equivalent to problem 114 with n=30.

STEP 3

3 or more As

All else Os, except for are L.

$\binom{\text{leftovers}}{2}$

Check for n=4.

+ 3 As in a row AND 2 Ls A A A O L A O L A ... $\binom{27}{2}$ 27 spots

Vanity Fair.

Faded by Readiness

Vierdumhuffer.